

Backward Induction Revisited

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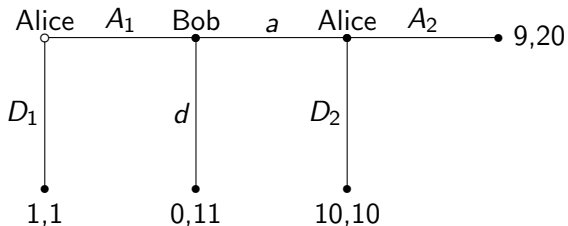
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Introduction I



Introduction II

- how to account for **hypothetical reasoning** in epistemic characterizations of Backward Induction?
- standard representation: **belief revision** in models for dynamic games
- making it explicit: re-defining strategy in terms of **conditionals**

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The Aumann-Stalnaker Debate

- Aumann (1995): **static** epistemic models $\mathcal{A} = (\Omega, s, (\mathcal{P}_i)_{i \in I})$
 - *CKR* implies *BI*
- Stalnaker (1998): **dynamic** epistemic models $\mathcal{S} = (\Omega, s, (Q_i, P_i)_{i \in I})$
 - Aumann's model cannot account for **belief revision** upon surprise information
 - implicit strong **epistemic independence** assumption:
no information about earlier actions of some player j are relevant to i 's beliefs about any later actions of j .
 - **Aumann's fallacy** of conflating two kinds of ifs:
 - 1 *If Alice had a second move, then she would choose D_2 .* (**causal if**)
 - 2 *If Alice has a second move, then she will not choose D_2 .* (**epistemic if**)

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Towards a New Definition of Strategy I

- making it explicit by moving the hypothetical reasoning (**conditionals**) to the surface (**strategy**)
- standard definition of strategy for player i as a **function**:

$$s_i : X_i \rightarrow A(X_i)$$

- a new definition of strategy for player i as a **set of subjunctive conditionals** of the form:

If play had reached x_i , then I would choose a .

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Towards a New Definition of Strategy II

- time: first **reasoning**, then **decision-making**
 - at t , the players reason about hypothetical situations by considering **subjunctive** conditionals
 - at $t + 1$, s represents the players' choices, and the strategy profile space is partitioned into **strict** and **counterfactual** conditionals
- possible world **semantics** for strategy
 - selection functions $f_i : \Omega \times X_i \times A_i \rightarrow \Omega$ characterize **closest world** in which play does reach x_i and i chooses a by adjusting i 's beliefs
- cf. Halpern's (2001) use of selection functions $f_i : \Omega \times X_i \rightarrow \Omega$
 - **extended Aumann Structures** to accommodate belief revision

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Conclusion

- re-definition of **strategy** in terms of **subjunctive conditionals** to account for **hypothetical reasoning** in games
- **semantics**: it will be interesting to study how different philosophical theories of conditionals fare for the re-definition
 - sensitivity of solution concepts?
- introduction of **time** to distinguish reasoning and decision-making
 - implications for **ex ante** and **ex post** notions?

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Thank you!