

Representation Theorems: rules and roles

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Decision, Games and Logic Workshop

Representation Theorems

Rules

Representation Theorem : Preference order on acts / propositions \rightarrow probability and utility / desirability functions

$$f \preceq g \text{ iff } \sum_s p(s) \cdot u(f(s)) \leq \sum_s p(s) \cdot u(g(s))$$

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(Implicit) rule:

- ▶ One set of acts / propositions
- ▶ One preference order

No reference to / consideration of **other** situations, with different sets of acts / propositions, and thus a different preference order.

Representation Theorems

... and Roles

Straw poll on roles:

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- Justificatory** Mmmm...

Example

State-dependent utility

Challenge

Axioms for state-*independent* utility do not appear hold in some situations

Eg. situations involving life or death

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Double bind (first horn)

	Successful	Not successful
Bet on success	\$100	\$0
Bet on failure	\$0	\$100

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State-dependent utility

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Axioms for state-*independent* utility do not appear hold in some situations

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Use representations involving state-dependent utilities:

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Representation theorems proposed which,

- ▶ given a preference relation on a set of acts, satisfying particular axioms,
- ▶ yield a (unique) probability and state-dependent utility representing the preferences.

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The utilities in this situation depend on the states.

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- ▶ is this an accurate enough description of his mental state?
- ▶ does this capture all we'd expect from a “rational” agent?
- ▶ Humph...

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In the elicitation of beliefs, desires, etc., one *should* make reference to other situations (be they counterfactual or subsequent).

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there exists representations of the preference relations in terms of “consistent” probability and utility functions.

“Consistent” = same probabilities / utilities in any situation with these states / consequences.

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If a bookmaker offered the agent certain acts, he would not trust him

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