

# ***Coordination with Intentions***

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Joint work with Martin van Hees (Groningen)

# Hi-Lo Games

	$t_1$	$t_2$
$s_1$	2, 2	0, 0
$s_2$	0, 0	1, 1

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2. Pre-play communication and correlated beliefs [Aumann, 1987]
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Maybe a missing item...

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6. **Intentions** also anchor coordination [Harman, 1976], [Bratman, 1987], [Velleman, 2003], [Wallace, 2006]

# Coordination with intentions

$i \setminus j$	$t_1$	$\dots$	$t_m$
$s_1$	$s_1, t_1$	$\dots$	$s_1, t_m$
$\vdots$			
$s_n$	$s_n, t_1$	$\dots$	$s_n, t_m$

- A **strategic game with intentions**  $G$  is a tuple  $G = \langle I, \{S_i, l_i\}_{i \in I}, \pi \rangle$ .
  - An **intention set**  $l_i$  is a set of strategy profiles.

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    - The **strategy choice** and **intention** of  $i$  at  $w$  are noted  $\sigma_i(w)$  and  $\iota_i(w)$ .

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I.e. the players know their own **strategies** and their own **intentions**.

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The strategy choice at a state  $w$  of a player  $i$  is **intention compatible** when :

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Let  $B_i$  be the set of highest-valued profiles for player  $i$ .  
The intention of  $i$  is **maximization compatible** at a state  $w$  iff

- $\iota_i(w) \subseteq B_i$ .

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# Connections with other explanations

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**Lemma** (van Hees & Roy, manuscript). *For any strategic **game of common interest** [Aumann and Sorin, 1989] with intentions  $G$  and strategy profile  $\sigma$ , if each  $\iota_i$  is **Nash equilibrium** compatible then the following are equivalent:*

- $\bigcap_i \iota_i = \{\sigma\}$
- $\sigma$  is the “Stackelberg solution” [Colman and Bacharach, 1997] of  $G$ .

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- How does such coordination relates to **other explanations**, e.g. correlated beliefs?
- What are **belief-based** sufficient conditions for coordination with intentions?
- The “own action problem” [Bratman, 2006]. How can players intend **profiles** and not only strategies?

# References

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